MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
THE NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
“Igor Sikorsky Kyiv Polytechnic Institute”
INSTITUTE OF MECHANICAL ENGINEERING

APPROVED BY
Director of the Institute
of mechanical engineering
_____ Mykola Bobyr
«____»_____ 2020

HIGHER MATHEMATICS

CURRICULUM OF DISCIPLINE

The first bachelor level of higher education

Specialty 131 "Applied Mechanics"

Profile program Manufacturing Engineering

Approved by the methodical commission of
the Institute of mechanical engineering
Protocol dated _____ 2020 № ___
Head of the methodical commission
_____ (Oleksandr Okhrimenko)
«____»_____ 2020.

Kyiv – 2020

DEVELOPER OF THE CURRICULUM:

Associate Professor, PhD Ganna Zhuravska _____

The curriculum is approved at the meeting of the department of Mathematical Physics of the Faculty of Physics and Mathematics of “Igor Sikorsky Kyiv Polytechnic Institute”

Protocol dated _____2020 № ___

Head of the Department _____Volodymyr Gorbachuk

«____» _____2020.

Introduction

Curriculum of the discipline "Higher Mathematics" is based on profile programs "Manufacturing Engineering" for the first bachelor level of higher education of specialty 131 "Applied Mechanics".

The discipline belongs to the cycle of general training.

The status of the discipline is compulsory.

The volume of the discipline is the 17 ECTS credits.

Interdisciplinary connections:

The discipline "Higher Mathematics" is closely linked with other disciplines of the natural sciences cycle such as Physics, Engineering Graphics, Computer Science, Electrical Engineering and Electronics; must precede and provide the following disciplines in the program of professional and practical training of a specialist: Engineering Mechanics, Strength of Materials, Technology of Structural Materials, Theory of Mechanisms and Machines, Theoretical Mechanics, Mathematical Modeling ect.

The subject of the discipline "Higher Mathematics" includes the basic methods of such sections of mathematics as the theory of limits, the differential and integral calculus of functions of one and several variables, differential equations, numerical series and series of functions, vector calculus and the theory of functions of the complex variable.

1. Learning Objectives and Tasks of the Discipline

The purpose of the discipline is to form students' abilities:

- thinking logically and flexibly;
- using methods of mathematics in engineering calculations;
- analyzing the results obtained;
- generalizing, setting the goal and choosing ways to solve it;
- independently using and studying literature on mathematics.

Objective of the discipline is shaping the following knowledge and skills of students.

Knowledge:

- basic definitions of the functions of one variable (domain, range, basic characteristics of functions, basic elementary functions and their graphs)
- the principles of differential calculus of functions of one variable (limit of a numerical sequence, limit of a function, concept of infinitesimals, concept of continuous function, the points of discontinuity of function, the tangent and the normal line to the curve, the derivative and the differential of a function, asymptotes, applications of derivatives: monotonicity, extrema and concavity, L'Hospital's rule)
- the bases of differential calculus of functions of many variables (partial derivatives, full differential, surfaces, extrema of the function of two variables);

- the bases of the integral calculus of functions of one and many variables (indefinite integral, properties of an indefinite integral, a table of integrals, methods of integration, definite integral and its geometric interpretation, fundamental theorem of calculus, improper integrals, double and triple integrals, methods of calculation in different coordinate systems, line and surface integrals, application of all kinds of integrals to problems of geometry and physics, vector calculus);
- the fundamentals of the theory and practice of ordinary differential equations (differential equations of the first order, separable, homogeneous and Bernoulli differential equations, the initial value problem, differential equations of higher orders, linear constant coefficients homogeneous and non homogeneous differential equations of higher orders, reduction of order);
- the bases of the theory of series (concept of numerical series, convergence and divergence of a series, absolute and conditional convergence for alternating series, function series, power series, interval of convergence, Taylor series, Fourier series, applications of series);
- the fundamentals of the theory of functions of a complex variable and an operational calculus (concepts, derivatives, and integrals, Taylor series and Laurent series of functions of a complex variable, Laplace and Fourier transformations, application of the Laplace transformation for solving ordinary differential equations and systems of such equations).

Skills:

- to know of graphs of basic elementary functions, to perform transformations of graph, to find the roots of polynomials, factoring polynomials, to perform operations on complex numbers;
- to find the limits of sequences and functions, to compare infinitesimals, to investigate the function for continuity, to classify points of discontinuity and asymptotes of the function, to find derivatives and differentials of functions of one variable, to apply the derivative and differential calculus, to make the investigation of functions, to use of L'Hospital's rule;
- to find partial derivatives and differentials for a function of several variables; to apply partial derivatives; to find extrema of a function of two variables; to find the gradient and directional derivative of a scalar field;
- to find indefinite integrals of rational, fractional-rational, trigonometric, irrational functions by basic methods of integration;
- to calculate of definite integrals, double, triple, line and surface integrals in different systems of coordinates, to apply of all kinds of integrals;
- to find general and partial solutions of differential equations; to find general and partial solutions of systems of linear constant coefficients equations;
- to study numerical, functional and power series, to use Taylor series and Fourier series, to orientate in the fields of their application;
- to find derivative and to calculate integrals of a functions of a complex variable, to represent a function in the form of Taylor or Laurent series, to use Laplace transforms for solving ordinary differential equations and systems of such equations.

2. Content of the Discipline

Part 1. Theory of Limits.

Topic 1.1. The Concept of Sets. Binary Operations with Sets. The Set of Real Numbers and Their Properties. The Absolute Value of a Real Number.

Topic 1.2. The Concept of Function. Ways of Representing Functions. Properties of Functions. The Inverse Function. Composite Function. Basic Elementary Functions and Their Graphs.

Topic 1.3. Numerical Sequence. Basic Concepts. The Limit of a Numerical Sequence. An Infinitely Large Variable.

Topic 1.4. The Limit of a Function. One-sided Limits. Infinitely Large Functions. Infinitesimals and Their Properties.

Topic 1.5. Basic Theorems on Limits. The Limit of the Function $\frac{\sin x}{x}$ as $x \to 0$. Number $\mathbf{e}$. Natural Logarithms.

Topic 1.6. Comparing of Infinitesimals. Equivalent Infinitesimals and Their Applications.

Topic 1.7. Continuity of Functions. Classification the Points of Discontinuity of a Function. Basic Theorems on Continuous Functions.

Topic 1.8. Certain Properties of Continuous Functions.

Part 2. Differential Calculus of a Function of one variable.

Topic 2.1. Definition of Derivative. Mechanical, Physical and Geometric Meanings of the Derivative. The Equations of a Tangent and a Normal Lines. Differentiability of Functions. Basic Rules of Differentiation: Derivative of a Sum, a Product, and a Quotient. The Derivative of a Composite Function (Chain Rule). The Derivative of an Inverse Function.

Topic 2.2. Derivatives of Basic Elementary Functions: Power Function, Exponential Function, Logarithmic Function, Trigonometric Functions, Inverse Trigonometric Functions, Hyperbolic Functions. The Table of Derivatives.

Topic 2.3. The Derivative of a Function Represented Parametrically. The Derivative of an Implicit Function. The Logarithmic Differentiation.

Topic 2.4. Derivatives of Higher Orders. Mechanical Significance of the Second Derivative. Derivatives of Higher Orders of Implicit Functions and of Functions Represented Parametrically.

Topic 2.5. The Differential and Its Geometric Significance. Differentials of Higher Orders.

Topic 2.6. Basic Theorems of the Differential calculus. A Theorem on the Roots of a Derivative. A Theorem on Finite Increments. A Theorem on the Ratio of the Increments of Two Functions. The L'Hospital's Rule.

Topic 2.7. Taylor's and Maclaurin's Formulas. Maclaurin's formulas for Basic Elementary Functions.

Topic 2.8. The Monotonicity of a Function. Local Extrema of a Function. The Largest and Smallest Values of a Function.

Topic 2.9. Concavity of a Curve. Points of Inflection. Asymptotes. The General Plan for Investigating Functions and Constructing Graphs.

Part 3. Integral Calculus of Functions of one variable.

Topic 3.1. The Concept of an Antiderivative. The Indefinite Integral and its properties. Table of Integrals.

Topic 3.2. Integration by Substitution. Integration by Parts.

Topic 3.3. Integration of Rational, Irrational and Trigonometric Functions.

Topic 3.4. The Definite Integral. Properties of the Definite Integral. Fundamental Theorem of Calculus (Newton-Leibniz Formula). Techniques of Evaluating Definite Integrals (Integration by Parts; Integration by Substitution). Improper Integrals.

Topic 3.5. Application of the Definite Integral. The Area of a region. The Volume of a Solid of Revolution. The Arc length. The Surface of a Solid of Revolution.

Part 4. Differential Calculus of Functions of Several Variables.

Topic 4.1. The Concept of a Function of Several Variables. Limit and Continuity of a Function of Several Variables.

Topic 4.2. Partial Derivatives of a Function of Several Variables. Partial Derivatives of Higher Orders. The Partial Derivatives of a Composite Function.

Topic 4.3 Differentials of a Function of Several Variables and Their Properties. The Partial Derivatives of an Implicit Function.

Topic 4.4. The Tangent Plane and the Normal Line to a Surface. Taylor's Formula for the Function of Two Variables. Local Extrema of a Function of Two Variables. The Largest and Smallest Value of a Function of Two Variables in the Region.

Part 5. Multiple Integrals.

Topic 5.1. The Concept of a Double Integral. Properties. Calculating Double Integral in Cartesian Coordinates.

Topic 5.2. Changing Variables in a Double Integral. Double Integral in Polar Coordinates. Application of a Double Integral to Problems of Geometry and Mechanics.

Topic 5.3. The Concept of a Triple Integral. Properties. Calculating Triple Integral in Cartesian Coordinates.

Topic 5.4. Changing Variables in a Triple Integral. Calculating Triple Integral in Cylindrical and Spherical Coordinates. Application of the Triple Integral to Problems of Geometry and Mechanics.

Part 6. Line Integrals.

Topic 6.1. The Concept of Line Integrals with Respect to Arc Length. Properties, Calculating and Application.

Topic 6.2. The Concept of Line Integrals of Vector Fields. Properties, Calculating and Application.

Topic 6.3. Green's formula. Independence of Path.

Part 7. Surface Integrals.

Topic 7.1. The Concept of Surface Integrals. Properties. Oriented Surfaces.

Topic 7.2. Calculating Surface Integrals. Application.

Topic 7.3. Ostrogradsky's Formula. Stokes' formula.

Part 8. Vector calculus.

Topic 8.1. Basic Concepts of Vector calculus. Scalar Field and Its Properties. Gradient. Directional Derivative.

Topic 8.2. Vector field. Flux of a Vector Field. Divergence. Divergence Theorem.

Topic 8.3. Circulation around the closed curve. Curl.

Topic 8.4. The Hamiltonian Operator and Certain Applications of It.

Part 9. First-Order Differential Equations.

Topic 9.1. Basic Concepts and Definitions. The Initial Value Problem. Differential Equations with Separable Variables. Homogeneous Differential Equations. Exact Differential Equations.

Topic 9.2. First-Order Linear Differential Equations. Bernoulli's equation.

Part 10. Higher-Order Differential Equations.

Topic 10.1. Basic Concepts and Definitions. Some Types of Second-Order Differential Equations Reducible to First-Order Equations.

Topic 10.2. Second-Order Homogeneous Linear Differential Equations. Functions that Are Linearly Independent on an Interval. Wronskian. Fundamental System of Solutions.

Topic 10.3. Second-Order Homogeneous Linear Differential Equations with Constant Coefficients. Characteristic Equation. General solution.

Topic 10.4. Second-Order Nonhomogeneous Linear Differential Equations. The Method of Variation of Arbitrary Constants.

Topic 10.5. Second-Order Nonhomogeneous Linear Differential Equations with Constant Coefficients and Special Right-Hand Side.

Topic 10.6. Systems of Differential Equations: Basic Concepts and Definitions. Systems of Linear Differential Equations with Constant Coefficients.

Part 11. Numerical Series.

Topic 11.1. Basic Concepts and Definitions of Numerical Series. Properties of Numerical Series. Necessary Condition for Convergence of a Series.

Topic 11.2. Comparing Series with Positive Terms. Tests of Convergence.

Topic 11.3. Alternating Series. Leibniz's Theorem. Absolute and Conditional Convergence.

Part 12. Functional Series.

Topic 12.1. The Concept of a Functional Series. Domain of Convergence. Uniform Convergence of a Functional Series. The Continuity of the Sum of a Functional Series. Integration and Differentiation of Functional Series.

Topic 12.2. Power Series. Interval of Convergence. Properties of Power Series.

Topic 12.3. Taylor's and Maclaurin's Series. Examples of Expansion of Functions in Series.

Topic 12.4. Application of Power Series.

Part 13. Fourier series.

Topic 13.1. The Concept of a Fourier Series. Expansion of Functions in Fourier Series. Sufficient Conditions for Expandability.

Topic 13.2. Fourier Series for Even and Odd Functions. Fourier Series for Periodic Functions.

Topic 13.3. Fourier Series for Functions Defined on an Arbitrary. Application of Fourier Series.

Part 14. Elements of the Theory of Functions of a Complex Variable.

Topic 14.1. Complex Numbers and Operations on Complex Numbers. The Concept of a Function of a Complex Variable. Limits and Continuity.

Topic 14.2. Differentiating the Function of a Complex Variable. Cauchy-Riemann Conditions. Analytic Functions and Their Properties.

Topic 14.3. The Geometric Meaning of the Derivative of a Function of a Complex Variable. The Concept of a Conformal Mapping.

Topic 14.4. An Integral with Respect to a Complex Variable. Definition and Properties. Cauchy's Integral.

Topic 14.5. Power Series on the Complex Plane. Taylor's and Laurent Series. Expansion of an Analytic Function in a Laurent Series.

Topic 14.6. A Classification of the Isolated Singular Points of a Single-Valued Analytic Function. The Residue of an Analytic Function at an Isolated Singularity. Application.

Part 15. Operational calculus.

Topic 15.1. Definition of the Laplace Transformation. Laplace Transformations of Some Elementary Functions.

Topic 15.2. The Main Properties of Laplace Transformation.

Topic 15.3. Determining the Original Function from the Transform. Mellin's formula.

Topic 15.4. Solving Problems for Linear Differential Equations and Systems Linear Differential Equations of by the Operational Method.

Topic 15.5. Fourier Integral and Fourier Transformation. Complex Form of a Fourier series.

3. Learning Activities and Teaching Methods

An important form of learning mathematics is the independent work of students, such as homework to each topic of the course and individual tasks.

Individual tasks are designed in the form of a list containing the most important problems of the credit module. They are aimed at broadening and deepening theoretical knowledge of students and promoting independent solving of specific problems.

Basic teaching methods:

1. Solving examples and tasks during practical trainings through active and collective learning, part-search and research methods;
2. As an addition to traditional training activities there are proposed such activities as Internet mailing, Distance and online learning, online testing ect.

4. Assessment of learning outcomes

Execution ongoing control and assessment of student's knowledge are made due to tests and individual tasks, the purpose of which is to identify the level of assimilation of relevant topics.

The final test is an exam in the end of each semester. The 100-points rating system and the university scale are used to evaluate the results of the training.

5. Readings

1. N.Piscunov Differential and Integral Calculus/ N.Piscunov – Mir Publisher, Moscow, 1966 – 895 p.

2. H. Jerome Keisler Elementary Calculus: an Infinitesimal Approach/ H. Jerome Keisler – On-line Edition. 2000
https://www.math.wisc.edu/~keisler/calc.html

3. Jeffrey R. Chasnov Introduction to Differential Equations. Lecture notes for MATH 2351/2352 / Jeffrey R. Chasnov – The Hong Kong University of Science and Technology, 2016 – 147p.
http://www.math.ust.hk/~machas/differential-equations.pdf

4. A.G.Sveshnikov The Theory of Functions of a Complex Variable/ A.G.Sveshnikov, A.N.Tichonov – Mir Publisher, Moscow, 1982 – 344 p.
https://archive.org/details/SveshnikovTikhonovTheTheoryOfFunctionsOfAComplexVariable

5. B.A. Fuchs Functions of a Complex Variables and Some of Their Applications/ B.A. Fuchs, B.V. Shabat – Elsevier, 1964 – 458 p.– ISBN 978-0-08-009404-5
https://www.sciencedirect.com/science/book/9780080094045

6. Informational Support

1. http://tutorial.math.lamar.edu
2. https://www.khanacademy.org
3. https://www.vitutor.com